Stretching of Heterogeneous Skewed Plates

The boundary conditions to be considered now are

$$\text{at } x = \pm 1: \quad M_x = 0, N_x = 0, N_{xy} = 0 \tag{18}$$

$$\text{at } y = \pm {}^1/_2 c - xtg\alpha: \quad M_n = 0, 21N_y = N, N_{xy} = 0 \tag{19}$$

$$\text{at the corners: } \quad M_{xy} + M_{nt} = 0 \tag{20}$$

Using the same procedure as before, we find for the constants in Eq. (5)

$$[k] = -N_y[\mathfrak{D}^{-1}] \times \begin{bmatrix} C_{xy}^* \\ C_{yy}^* \cos^2\alpha + C_{xy}^* \sin^2\alpha + 2C_{ys}^* \cos\alpha \sin\alpha \\ C_{ys}^* + (C_{xy}^* - C_{yy}^*)\cos\alpha\sin\alpha + C_{ss}^*(\cos^2\alpha - \sin^2\alpha) \end{bmatrix} \tag{21}$$

Thus simple stretching of the plate is associated with a deflection pattern [Eqs. (5) and (21)]. Present results for pure bending and twisting include—as a special case previously obtained—expressions by Reissner[3] for homogeneous, isotropic, skewed plates.

It is noted that the solution [Eq. (21)] for the stretching problem and the cross effect in the stress-resultants-couples relations, observed in all three cases analyzed, do not appear at all in homogeneous plates.

References

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Slip Flow of an Ionized Gas Over a Sphere Carrying a Magnetic Dipole†

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In this note, the work of Barthel and Lykoudis[1] on the motion of a conducting gas about a magnetized sphere at low Reynolds number is extended to the case in which the effects of slip flow become important.

The ordinary magnetofluidmechanic approximations are used throughout; hence, the Navier-Stokes equations, with the added terms representing the ponderomotive force, are used. It is open to question whether the Navier-Stokes equations are valid at low densities where slip phenomena are important. However, there are enormous difficulties, as well as serious ambiguities, connected with the application of the Burnett, thirteen-moment, and other equations.[2]

The effects of slip are accounted for, following Maxwell, by assuming that the tangential velocity is proportional to the velocity gradient at the wall. The coefficient of proportionality is designated as the slip coefficient and is given the symbol ζ. It is related to the mean free path by the equation

$$\zeta = [(2 - \epsilon)/\epsilon]l$$

where l is the mean free path and ϵ is the fraction of molecules which are diffusely reflected from the wall (with zero average tangential velocity).

The problem of slip flow over a nonmagnetized sphere at low Reynolds numbers was solved by Basset;[3] therefore, his solution is used in calculating the ponderomotive force in the perturbation analysis similar to that applied in Ref. 1.* Small magnetic Reynolds numbers are assumed; hence, the undistorted magnetic field for a dipole is used in the calculation of the ponderomotive force.

Applying the same method of solution as used in Ref. 1, the result for the coefficient of drag on the sphere due to the viscous forces, pressure forces, and the ponderomotive force is**

$$C_D = C_{D_0}\left[1 + M\left(\frac{0.004 + 0.170\phi + 1.737\phi^2 + 5.879\phi^3 + 6.281\phi^4}{0.573 + 8.975\phi + 47.429\phi^2 + 103.829\phi^3 + 80.571\phi^4}\right)\right] \tag{1}$$

where

$$\phi = \zeta/a = [(2 - \epsilon)/\epsilon](l/a)$$

and $M = \sigma B_0{}^2 a^2/\mu$ is the square of the usual Hartmann number, σ the electrical conductivity, B_0 the magnetic flux at the poles, a the radius of the sphere, and μ the dynamic viscosity.

In the nonmagnetic case C_{D0} represents the drag coefficient allowing for slip and is given by

$$C_{D0} = (12/R)[(1 + 2\phi)/(1 + 3\phi)]$$

where R is the kinematic Reynolds number. When $\phi = 0$ (no slip), Eq. (1) reduces to

$$C_D = (12/R)[1 + (M/150)]$$

which concurs with the first-iteration result of Ref. 1.

The figure shows the variation of C_D/C_{D_0} with ϕ (essentially the Knudsen number) at different Hartmann numbers. It can be readily seen that, for large values of the slip coefficient, the effect of the magnetic field upon drag is quite pronounced. This is due to the fact that, when appreciable velocities are allowed at the surface of the sphere, the ponderomotive forces in the vicinity of the surface can become quite large, since the magnetic field is strongest there.

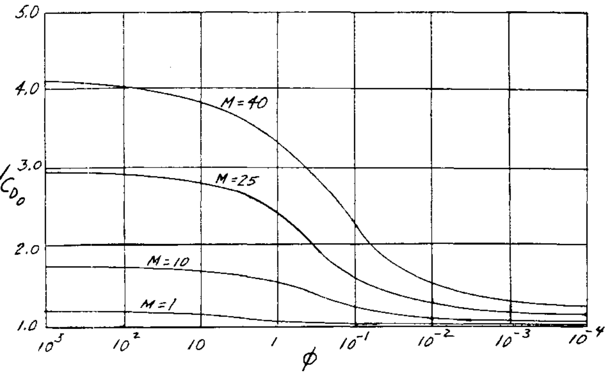

Fig. 1. Variation of drag with slip coefficient.

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** The details of these and all other calculations can be found in Ref. 4.

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Determination of Heliocentric Elliptic Orbit

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IN THIS PAPER a method of determining heliocentric elliptic orbits is presented. The procedure makes use of the following assumptions.

Symbols

ϕ	= a differentiable scalar field
∇	$= \frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}$
$\mathbf{r}_{1-2}$	= radius vector from heliocenter to perihelion
$\mathbf{r}_{1-3}$	= radius vector from heliocenter to a point on orbit
$\hat{\mathbf{P}}, \hat{\mathbf{Q}}, \hat{\mathbf{W}}$	= unit vectors along x_w, y_w, z_w axes of orbit plane, respectively
e	= eccentricity
$\hat{\mathbf{x}}$	= unit vector along x-axis
β	= angle between orbit plane radius vector and major axis of ellipse
x_w, y_w	= x and y axes of orbit plane
a, b	= major and minor axes of elliptic orbit
r_i	= $\lvert\mathbf{r}_{1-2}\rvert$ and $\lvert\mathbf{r}_{1-3}\rvert$
E	= eccentric anomaly
M	= mean anomaly

(1) The coordinates (x_1, y_1, z_1) of position of the radius vector $\vec{\mathbf{R}}_1$ from geocenter to heliocenter are known.

(2) The coordinates (x_2, y_2, z_2) of position of the radius vector $\vec{\mathbf{R}}_2$ from geocenter to perihelion are known.

(3) The coordinates (x_3, y_3, z_3) of position of the radius vector $\vec{\mathbf{R}}_3$ from geocenter to any point on the orbit of the planet are known.

Knowing the positions of these three radius vectors, we take the following steps out to find the various orbital elements.

(1) The angle or slope that the major orbital axis makes with x-axis of $z = 0$ plane is determined as follows:

The points of projections of (x_1, y_1, z_1) and (x_2, y_2, z_2) are $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$, respectively.

The slope m_{1-2} of a line through $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ is

$$m_{1-2} = (y_2 - y_1)/(x_2 - x_1) \tag{1}$$

Similarly, the slope m_{1-3} of a line through $(x_1, y_1, 0)$ and $(x_3, y_3, 0)$ is

$$m_{1-3} = (y_3 - y_1)/(x_3 - x_1) \tag{1a}$$

(2) The equation of the orbit plane through (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3) is found by the familiar use of

$$AX + BY + CZ + D = 0 \tag{2}$$

Substituting the coordinates of these points into Eq. (2), the values of the constants can be determined and, hence, the equation of the plane can be obtained. From this, one can write

$$\phi(x, y, z) = C \tag{3}$$

Again, the equation of the plane at the geocenter is

$$z = 0 \tag{4}$$

By transferring Eq. (4) to the heliocenter, one can write for the plane

$$\phi(z) = C \tag{5}$$

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When the origin is transferred to the heliocenter, the axes remaining parallel to their original positions, the coordinates of the perihelion become $(x_2 - x_1)$, $(y_2 - y_1)$, and $(z_2 - z_1)$, and thus

$$\lvert\mathbf{r}_{1-2}\rvert = q = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{6}$$

and the length of the projection of $\mathbf{r}_{1-2}$ on $z = C$ is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{7}$$

Similarly, for point (x_3, y_3, z_3),

$$\lvert\mathbf{r}_{1-3}\rvert = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2} \tag{8}$$

(3) The vector perpendicular to orbit plane at the heliocenter, from Eq. (3), is given by

$$\nabla(\phi_1) = \nabla\phi(x, y, z) \tag{9}$$

Thus, the unit vector

$$\hat{\mathrm{w}} = \frac{\nabla\phi(x, y, z)}{\lvert\nabla\phi_1\rvert} \tag{10}$$

and the vector perpendicular to the plane $z = C$ at the heliocenter, from Eq. (5), is given by

$$\nabla(\phi_2) = \nabla\phi(z) \tag{11}$$

The angle between $\nabla(\phi_1)$ and $\nabla(\phi_2)$ is

$$\theta = \cos^{-1}\frac{\nabla(\phi_1)\cdot\nabla(\phi_2)}{\lvert\nabla(\phi_1)\rvert\,\lvert\nabla(\phi_2)\rvert} \tag{12}$$

This is also the angle of inclination.

After bringing the x-axis in line with Eq. (7) by using Eq. (1), the vector through the node is

$$\nabla(\phi_1)x\nabla(\phi_2)$$

The unit vector through the node

$$\hat{\mathbf{N}} = \frac{\nabla(\phi_1)x\nabla(\phi_2)}{\lvert\nabla(\phi_1)x\nabla(\phi_2)\rvert} \tag{13}$$

The angle between the node and the x-axis is

$$\psi = \cos^{-1}\frac{\hat{\mathbf{N}}\cdot\hat{\mathbf{x}}}{\lvert\hat{\mathbf{N}}\rvert\,\lvert\hat{\mathbf{x}}\rvert} \tag{14}$$

$$\hat{\mathbf{x}} = (\hat{\mathbf{x}}\cdot\hat{\mathbf{P}})\hat{\mathbf{P}} = \hat{\mathbf{P}}\cos\alpha$$

therefore,

$$\hat{\mathbf{P}} = \hat{\mathbf{x}}/\cos\alpha \tag{15}$$

where α is the angle between Eqs. (6) and (7). The unit vector

$$\hat{\mathbf{Q}} = \hat{\mathbf{P}}x\hat{\mathbf{W}} \tag{16}$$

From Eq. (6) and conic relationships,[4] the semilatus rectum is

$$p = 2q = 2\lvert\mathbf{r}_{1-2}\rvert = b^2/a \tag{17}$$

and

$$e = p - r_i/x_{w_i} \tag{18}$$

Using Eqs. (1), (1a), (6), and (8),

$$x_{w_i} = r_i\cos\beta \tag{19}$$

$$a = q/(1 - e) \tag{20}$$

From Eq. (144) of Ref. 4,

$$\cos E_i = e + x_{w_i}/a$$

and

$$\sin E_i = y_{w_i}/a\sqrt{(1 - e^2)}$$

Here, $i = 1$ and 2.

From Kepler's equation [Eq. (4) of Ref. 4],

$$M_1 = E_1 - e\sin E_1 = n(t - T)_1, \quad 1 \rightarrow 2$$

and knowing[4]

$$n = k\sqrt{m_s + m_P}\,a^{-3/2}$$

where $k\sqrt{m_s + m_P}$ = constant, the time interval